Classwork 15: Instrumental Variables (part 1)

EC 421

2022-11-19

Part 1: Consistency of IV Proof

1a) In the workbook we saw that $plim(\hat{\beta_1}^{IV}) = \frac{Cov(\hat{x_i}, y_i)}{Var(\hat{x_i})}$.

Use the fact that $\hat{x_i} = \gamma_0 + \gamma_1 z_i$ to show that $plim(\hat{\beta_1}^{IV}) = \frac{Cov(z_i, y_i)}{\gamma_1 Var(z_i)}$.

1b) Next show that: $plim(\hat{\beta_1}^{IV}) = \beta_1 + \frac{Cov(z_i, u_i)}{Cov(z_i, x_i)}$.

It may be helpful to use the formula derived in the previous problem, the fact that $y_i = \beta_0 + \beta_1 x_i + u_i$ (as long as the exclusion restriction holds, y_i is not directly a function of z_i), and this formula for γ_1 from the first stage: $\gamma_1 = \frac{Cov(x_i, z_i)}{Var(z_i)}$.

1c) Using the formula derived above: $plim(\hat{\beta_1}^{IV}) = \beta_1 + \frac{Cov(z_i, u_i)}{Cov(z_i, x_i)}$,

argue why the conditions for an instrument Z to be valid are the same as the conditions for $\hat{\beta_1}^{IV}$ to be consistent. We've used the exclusion restriction condition in the previous problem, but you should talk about the other two here.

Part 2: Supply and Demand

In chapter 10, we'll explore how IV can be used to estimate supply or demand curves under *Simultaneity Bias* (the fact that price and quantity is determined simultaneously in a market by supply and demand). To help us get started with that chapter, solve these problems:

2a) Solve for p and q

Demand: $q_i = \alpha_0 + \alpha_1 p_i + u_i$

Where u_i include effects of demand shifters like changes in consumer tastes, advertising campaigns, and changes to the prices of substitutes or complements.

Supply: $q_i = \beta_0 + \beta_1 p_i + v_i$

Where v_i include effects of supply shifters like weather shocks or other fluctuations in the costs of production.

Solve for p_i and q_i so that we get an equation for q_i that doesn't contain p_i, and an equation for p_i that doesn't contain q_i.

To check your answers, you should get that:

$q_i = \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 v_i - \beta_1 u_i}{\alpha_1 - \beta_1}$

$p_i = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{v_i - u_i}{\alpha_1 - \beta_1}$

2b) Show that Cov(p, u) $>$ 0 and Cov(p, v) $<$ 0.

You can assume that Cov(v, u) $= 0$.