Classwork 1 (analytical)

EC421

2022-07-19

In video set 1, we took the definition of the method of least squares and we derived that $\hat{\beta}_1 = \frac{\sum_i (x_i y_i) - n\bar{x}\bar{y}}{\sum_i (x_i^2) - n\bar{x}^2}$.

1) Show that $\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$.

2) Use the formula for $\hat{\beta}_1$ derived in 1) to show that $\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x}) y_i}{\sum_i (x_i - \bar{x})^2}$.

3) Use the formula for $\hat{\beta}_1$ in 2) to show that $\hat{\beta}_1 = \beta_1 + \frac{\sum_i (x_i - \bar{x}) u_i}{\sum_i (x_i - \bar{x})^2}$.

Hint: note that the left hand side is the estimate for β_1 and the right hand side includes the true value of β_1. These will not be exactly equivalent except by chance. You should start this problem by making a substitution for y_i, since $y_i = \beta_0 + \beta_1 x_i + u_i$. This will get the true β_1 and u_i into the equation.

Extra Credit: Numerical Example

Recall: $\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

Use those formulas to calculate $\hat{\beta}_1$ and $\hat{\beta}_0$ by hand. It may be helpful to draw a plot of the data and try to eyeball the line of best fit in order to double check that your answers make sense.

x	y
0	1
1	2
1	3
0	2