Classwork 1: Covariance and Correlation

Important formulas:

symbol	formula	Discrete RV Calculation
μ_X	Expected value $E[X]$	$\sum_{i=1}^n x_i p_i$
σ_X^2	$Var(X) = E[(X-\mu_X)^2]$	$\sum_{i=1}^n (x_i - \mu_X)^2 p_i$
σ_{XY}	$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$	$\sum_{i=1}^n (x_i - \mu_X)(y_i - \mu_Y) p_i$
ρ_{XY}	$Corr(X,Y) = \frac{\sigma_{XY}}{\sqrt{\sigma_X^2 \sigma_Y^2}}$	$\leftarrow$

In this classwork, you will demonstrate that covariance depends on units of measurement while correlation does not. To do this, consider the table below. It is a joint probability distribution between heights and weights: suppose people are only 120 lbs or 180 lbs, and also only 60 inches or 70 inches. The joint probability distribution says, for example, that someone is 60 inches and 120 lbs with a probability of 0.4.

It will probably be useful to calculate the marginal probabilities, which you can do by summing across the rows or columns. For instance, recognize that someone is 60 inches tall with a probability of $0.4 + 0.1 = 0.5$.

		height (inches)	
		60	70
weight (lbs)	120	0.4	0.1
	180	0.1	0.4

1. Show that $\mu_{height} = 65$ and that $\mu_{weight} = 150$. (2 points)

2. Show that $Cov(height, weight) = 90$. (2 points)

3. Show that when heights and weights are instead measured in centimeters and kilograms, the covariance between those two variables is not 90, it's closer to 103.6. (2 points) For reference:
   120 lbs = 54.4 kg
   180 lbs = 81.6 kg
   60 inches = 152.4 cm
   70 inches = 177.8 cm

4. Show that the correlation between height and weight is 0.6, whether you measure height and weight in inches and lbs *or* in centimeters and kilograms. (4 points)