Cost Minimization Review: Extra Credit

Fill out and turn this worksheet in before class on Tuesday, October 29th to earn up to 2 points added on to your Quiz 7 grade (a $5/3 = 167\%$ is possible for that assignment).

1 Cost Minimization Problem

1. A firm wants to minimize its total costs (TC) while producing 100 units of output. Suppose the firm's production function is given by:

$$Q = 2K^{0.5}L^{0.5}$$

Where:

- K represents capital
- L represents labor

The costs of the inputs are:

- p_K is \$3 per hour
- p_L is \$12 per hour

Objective: Find the minimum-cost way to produce 100 units of output. That is:

$$\text{minimize } TC = 3K + 12L \quad (1)$$
$$\text{s.t. } 100 = 2K^{0.5}L^{0.5} \quad (2)$$

Follow steps a-d to solve this cost minimization problem.

(a) Take the production function and solve for L in terms of K. That is, show that $L = \frac{2500}{K}$.

(b) Express TC solely in terms of K by plugging in $L = \frac{2500}{K}$ into the TC function. That is, show that $TC = 3K + \frac{30000}{K}$.

(c) Find the value of K that minimizes TC by taking the derivative of TC (use the equation from part b) with respect to K, setting it equal to 0, and solving for K. You should find that K = 100.

(d) Find the corresponding value of L by plugging in K = 100 into the $L = \frac{2500}{K}$ equation.

(e) Show that the minimum total cost to produce 100 units of output is \$600.

The next 2 problems give you more practice with this procedure, but offering fewer and fewer hints.

2. Take $Q = 5K^{0.5}L^{0.5}$. If $p_K = 4$ and \$pL = 16, what is the lowest cost way for the firm to make 100 units of output?

   (a) Show that $L = \frac{400}{K}$, and use that to eliminate L in the total cost function.

(b) Show that the value for K where TC is minimized is $K^* = 40$.

(c) Show that $L^* = 10$.

(d) Show that the total cost for making 150 units (when cost minimizing) is \$320.

3. Take $Q = 16K^{0.8}L^{0.2}$, $p_K = 1$, and $p_L = 8$. Show that the lowest-cost way for the producer to make 256 units of output is to use 32 units of capital and 1 unit of labor, which costs \$40.

2 Cobb-Douglas Shortcut

A Cobb-Douglas production function has this form: $Q = Ax^a x^b$. If $a + b = 1$, we can use the Cobb-Douglas shortcut to solve the cost minimization problem quickly.

4. Consider again problem 1: $Q = 2K^{.5}L^{.5}$ where $p_K = 3$ and $p_L = 12$. The Cobb-Douglas shortcut says that when the producer is cost minimizing, they will spend a% of their total cost on capital and b% of their total cost on labor.

   (a) Use the fact that the producer's expenditure on labor is $p_L L$ and the producer's expenditure on capital is $p_K K$ to show that $L^* = TC/24$ and $K^* = TC/6$.

   (b) Plug in L^* and K^* to the production function, along with $Q = 100$, to show that the minimum total cost to produce 100 units of output is $600.

   (c) Given that minimum total cost, how much K and L does the producer use? Verify your answers by comparing to problem 1.

5. Consider problem 2: If $Q = 5K^{0.5}L^{0.5}$, $p_K = 4$, and $p_L = 16$, use the Cobb-Douglas shortcut to find the lowest cost way to produce 100 units of output.

6. Consider problem 3: If $Q = 16K^{0.8}L^{0.2}$, $p_K = 1$, and $p_L = 8$, use the Cobb-Douglas shortcut to find the lowest cost way to produce 256 units of output.